Plenary session: Visual Modelling with Logo

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Visual Modelling as a Motivation for Studying Mathematics and Art

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Abstract

The paper deals with the possibility of enriching the curriculum in mathematics, informatics and art by means of visual modelling of abstract paintings. The authors share their belief that in building a computer model of a construct the students gain deeper insight into the construct and are especially motivated to elaborate their knowledge in mathematics and informatics. This idea is demonstrated in the context of educating pre- and in-service teachers in mathematics and informatics at Sofia University "St. Kliment Ohridski".

The visual modelling of some paintings by de Stijl, Kandinsky, Sonia Delaune, Escher and Balkanski by means of identifying fundamental geometric elements and describing them by Logo procedures is discussed on the basis of the authors' experience in various settings in Bulgaria and the Republic of Macedonia.

Key words

learning style, elementary education, curriculum, teacher education

1. INTRODUCTION

Research in mathematics education in various countries shows that for many students in the middle grades mathematics is just a computational work (often only slight extension of what is covered in the earlier grades). They do not see it as exploratory, dynamic, evolving discipline but rather as a rigid, closed body of rules to be memorised.

Many projects in reshaping the curriculum are oriented to helping students broaden their perspective, view mathematics as an integrated whole and acknowledge its relevance and usefulness in and out of school. Furthermore, they emphasise on the investigation of mathematical connections so that students can apply mathematical thinking and modelling to solve problems that arise in other disciplines such as art, and value the role of mathematics in our culture and society.

Such goals can be achieved in various ways but we would like to concentrate on visual modelling in a computer environment as an activity that can motivate the study of both mathematics and art.

The project Visual modelling described below was an attempt to integrate ideas from art and Logo in order to motivate the study of geometry. It was experimented at Sofia University "St. Kliment Ohridski" in the pre- and in-service teacher education. The project was later on extended in the frames of some extracurricular activities for primary and middle school children.

2. THE IMPORTANCE OF STUDYING GEOMETRY

The study of geometry plays a special role in helping students represent and make sense of the world. Geometric models provide a perspective from which students can analyse and solve problems, and geometric interpretations facilitate the understanding of abstract (symbolic) interpretation. According to various national standards in grades 5-8 (e.g. [1]), the mathematics curriculum should include the study of the geometry in a variety of situations so that the students could:

- identify, describe, compare and classify geometric figures
- visualise and represent geometric figures
- explore transformations of geometric figures
- represent and solve problems using geometric models
- understand and apply geometric properties and relationships

2.1. Some problems in the study of geometric shapes

At the same time investigations in countries with good traditions in mathematics education show that students in these grades have problems with identifying of even such simple geometric shapes as square and triangle if their basis is not horizontal. The reason for this is sought for not only by mathematics educators but also by cognitive scientists and psychologists.

It has been found [2] that up-down axis is a powerful organiser of our sense of shape and form. If we rotate the outline of Africa for instance ninety degrees few people would recognise it (even if they tilt their heads). The mental representation of a shape does not just reflect its Euclidean geometry, which remain unchanged as a shape is turned (under rotation). It reflects the geometry relative to our up-down reference frame. The psychologist Irvin Rock (ibid) has found many other examples, including this simple one (Figure 1):

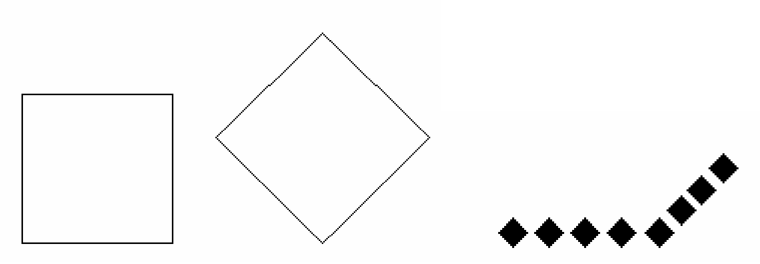

Figure 1. Square in different directions and groupings

People see the first two drawings as two different shapes, a square and a diamond, and the only difference is how they are aligned with respect to the viewer's up-and-down reference frame (and that difference is enough to earn them different words in the English language). It is even hard to see that the diamond is made of right angles. Finally, objects themselves can plot out reference frames. e.g. the shape at the bottom right of the third drawing flips between looking like a square and looking like a diamond, depending on where one mentally groups it.

Once the difficulties for identifying and classifying geometric shapes and getting familiar with their properties has objective nature, it becomes of a vital importance for mathematics educators to re-think not only the curriculum but also the strategies which could support the process of getting deeper understanding of this field.

2.2. The role of the computer environments

Computer environments of exploratory type (Comenius Logo [3] and Geomland [4] in our case) allow students to construct shapes on the screen and then flip, turn or slide them, to view them from a new perspective. Such explorations illuminate the concepts of congruence and similarity. Observing and learning to represent geometric figures in various positions by controlling the turtle help students to develop spatial sense.

Students' understanding of the angle properties of polygons and the concept of area can be additionally enhanced through explorations of tessellation with regular polygons.

3. VISUAL MODELLING

Exploring the properties of geometric shapes in a Logo environment becomes more exciting if made part of a visual modelling of some works of art. By building computer models of a given painting the students can gain deeper insight in its structure and motivation to elaborate their knowledge in mathematics and informatics. Let us consider some examples experimented in the context of educating pre- and in-service teachers and children alike

3.1. In the style of de Stijl and Kandinsky

The term De Stijl is usually associated with paintings showing only horizontal and vertical lines and planes of red, yellow and blue, with buildings resembling coloured blocks, and with artists so puritanical and dogmatic that they considered a diagonal line a mortal sin [5]. This stereotyped image is due mainly to Piet Mondrian's work and personality. However, it does not hold even for Mondrian, let alone for other artists who were involved with De Stijl, e.g. Van der Leck (Figure 2).

When analyzing an abstract painting from mathematical point of view it is interesting to discuss its basic elements and to classify them. Generating a specific shape, e.g. square, in the process of computer modelling from different initial positions and directions of the turtle enhances the understanding that this is the same shape. The next step is to create a procedure for all kind of rectangles (including the square). A further challenge for students is to generate all the elements with a single procedure for a filled-in parallelogram and thus to reinforce their understanding that the square and the rectangle are special cases of a more general class of figures.

Figure 2. Van der Leck, Composition 1918, no. 5

Next, paintings by Kandinsky containing a richer variety of shapes can be considered (Figure 3).

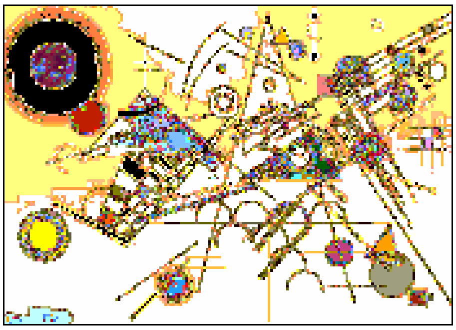

Figure 3. Kandinsky, Composition VIII, 1923

From an artistic point of view, though, the problem is not only to understand the elements of a composition, but also to understand its balance. This led to developing a Logo microworld in which it was easy to experiment with figures of various sizes, colours and degrees of complexity, i.e. to verify different definitions of balance.

In addition, the students could play with Kandinsky's ideas concerning the relation between geometric shape and colour and study the effect of both components in various combinations.

We could qualify the following factors as the most relevant ones in the study of an abstract painting [6]:

- The character of the objects and their composition in terms of clustering, overlapping, isolation, balance, relationship between size, shape and colour
- Main categories of the objects
- Establishing hierarchy related to the distance of the centre, the size, the colour, etc.
- Functional associations (which objects occur in combination in the work of a given author)

Products of the visual modelling should be judged with respect not only to the closeness between the original and the generated works but also to their potential to generate works bringing the spirit of the original together with new, unexpected ideas. After leaving the frames of the strict imitation we could be inspired by new combinations of forms and colours and could get new insight, which in turn could lead to new formalisation.

3.2. In the style of Sonia Delaunay

At first glance the fashion and geometry do not have much in common. But this is not what the great French artist from Russian origin Sonia Delaunay thought when designing scenery and costumes for the ballet of Diaghilev [7].

Figure 4. Dress designs by Sonia Delaunay

Here are some computer variations of these models (Figure 5):

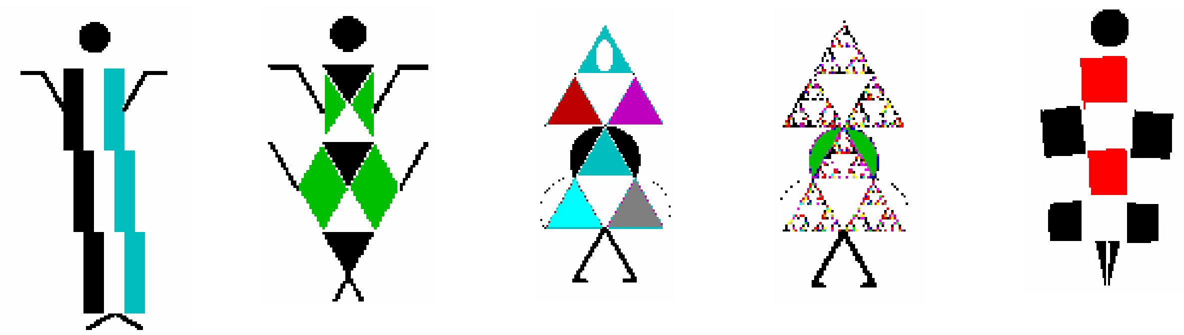

Figure 5. Computer variations of Sonia Delaunay's models

With all the models we start with a stylised version and then identify the geometric figures and transformation which could be used in order to create the closest possible computer version of the model. Various approaches are possible and one of the challenges is to use mathematics knowledge, which is accessible for younger students. Another goal is to motivate students to create their own informatics tools e.g. filled-in triangles, polygons and parallelograms. Once a geometric shape participates in more models, the procedure is generalised so as to produce shapes of various sizes. The filling of the shapes itself becomes much easier if some additional informatics tools are introduced (in this case operations for enclosing several variables into a list and processing data structures as commands).

In the Logo version we are using each turtle drawing can be represented in two ways [8]:

- procedurally (by a piece of Logo program), or
- declaratively (by a piece of data – a list of numbers which are interpreted as inputs of Logo commands. Such lists are called drawing lists.)

For example the following instruction is a procedural representation of a circle:

```
REPEAT 360 [FORWARD 1 RIGHT 1]
```

With an appropriate interpretation the same circle could be represented declaratively by the following drawing list:

```
[360 [1 1]]
```

A more general declarative description of a circle of arbitrary size can be done by a drawing list composed by the LIST operation:

```
LIST 360 LIST :S 1
```

Here the input :S stands for the length of the side of the 360-gon approximating the circle.

Similarly a filled-in parallelogram of sides :S1, :S2 and angle :A could be described declaratively by the following drawing list:

```
LIST 2 (LIST :S1 :A :S2 180-:A)
```

Interpreting certain lists of numbers as declarative representation of turtle drawings is very useful since it provides a convenient instrument for drawing polygon lines and filled-in polygons which are transparent with respect to the turtle state.

This allows the students to concentrate on the process of combining the shapes and on modifying appropriately the data of the lists so as to get suitably transformed shapes.

The stylised versions of Delaunay's models (the first three ones in Figure 5) are built of filed-in rectangles and triangles or figures which could be considered as their combinations.

The basic building block in the 3d model represents a filled-in triangle divided in 4 congruent triangles with the central one cut out. It could be generated in many ways. When considered as a step of the construction process of the Sierpinski gasket [9] it could be generated as a concrete level of a recursive procedure thus allowing for interesting variations (e.g. the 4th model in Figure 5).

In the last model we implement a new idea – to introduce after the geometric stylisation the effect of a free-hand drawing. This could be done by introducing a random effect - the rectangles are replaced by quadrilaterals with randomly chosen angles close to 90°.

Products of the visual modelling should be judged with respect not only to the closeness between the original and the generated works but also to their potential to generate works bringing the spirit of the original together with new, unexpected ideas. After leaving the frames of the strict imitation we could be inspired by new combinations of forms and colours and could get new insight, which in turn could lead to new formalisation.

3.3. In the style of Escher

Which shapes and figures when fitted together fill space completely and which leave unfilled spaces? Which combination of figures could be used? These topics in geometry provide an excellent opportunity for teachers to introduce important mathematical concepts and for students to experience creative interplay between mathematics and art. Various approaches are possible. For example, by repeated tracing of a regular polygon about a point so that the tracing coincide only along edges and do not overlap, and then by extending their tracings outward on the paper, students can discover whether the polygon might be used to form a tessellation of the plane. Or similar procedures could be implemented in a computer environment by means of the Logo turtle.

Out of this very informal experience a fundamental question arises: What shapes can be stacked together to fill space completely? This fundamental question is analogous to investigating the manner in which matter splits into atoms and natural numbers split up into products of primes [10]. A more specific situation to be considered is which regular polygons can be used to tile a plane? Once students have determined that only the equilateral triangle, square and hexagon can be used to form a tiling pattern, project work for groups of students could include exploring:

- the number and nature of semi-regular tilings using a combination of two or more regular polygons and where these patterns appear in their environment
- the existence of non-regular polygons that would serve as a fundamental tiling unit

The graphics work of the Dutch artist Escher and the creation of Escher-type tessellations deserves a special attention in this context [11].

Provoked by the fact that Arabian artists, because of their religion, used only geometric figures as tiling, Escher started to experiment with stylised models of people, animals or other objects from the real or fictional world (Figure 6). He often described regular divisions of the plane as the richest source of inspiration [12]. How did he succeed to create so complex elements of tessellation? This question is a good topic for discussion with the students. With small hints of the teachers they could realise that basically Escher uses regular polygons and by appropriate geometric transformations (symmetry, translation, rotation and optical reflection) he gets the wanted shapes.

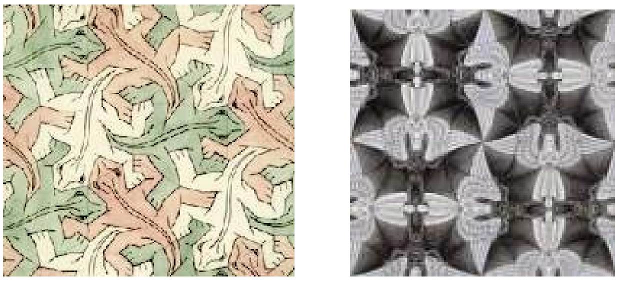

Figure 6. Escher Reptiles (left) and Angel&Devils (right)

Now, the following question arises: Can we play Escher?, i.e. how could we create stylised models of live beings or other objects that could represent a basic element of tessellation. This problem is known as Escherization problem and is appealing to many students since it provides an excellent setting for creative expression

An example of such type of modelling is seen in Figure 7 – by cutting a certain piece of the regular hexagon and using appropriate geometric transformations we get a reptile as a motive of tessellation. What is left is to determine the angle of rotation so as to generate the reptile tessellation.

Another interesting object for modelling is the fragment of the Escher's painting Angels & Devils. The tile he uses could by stylised and presented by a Logo procedure as in Figure 8. This time we start with a square, and then cut out a specific shape from it and perform a translation of the shape cut so as to get a figure of two bets and two angles.

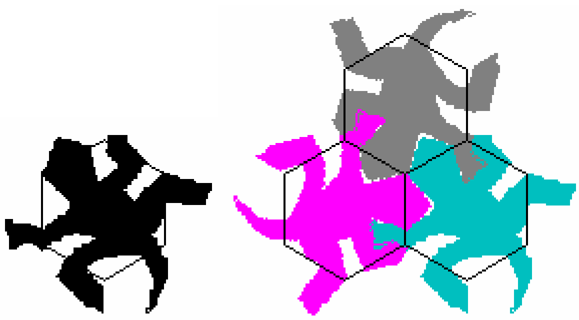

Figure 7. A Logo model of Escher's Reptiles

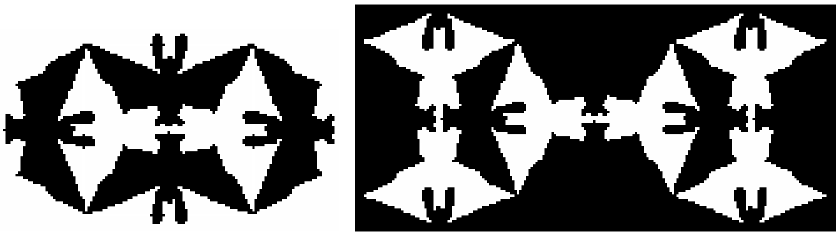

Figure 8. A Logo model of Escher's Angel&Devils

Interesting mathematical topics for consideration in this context are:

- Which is the minimal element producing the tile?
- What geometric transformations are needed to generate the tile (the whole tessellation)?

In order to optimise the drawing of the tile we can make use of the symmetry of the figure and create a procedure for the minimal element producing it. Then we can change the turtle direction appropriately with regard to the axis of symmetry and use a rotation to get the whole tile.

A very appropriate informatics tool for the purpose is to first develop a procedure for drawing by means of the arrow keys. This allows us to capture the finest details of the drawing. After the drawing is finished the procedure outputs a drawing list of the codes of the consecutive keys which have been pressed. Thus we get the declarative representation of the generating element. Next we can make a procedure for transforming it in a drawing list which generates the symmetric element with respect to the initial direction of the turtle.

Such interplay between mathematics and informatics ideas raises many interesting topics for discussions with in- and pre-service teachers: which informatics tools to provide as ready-made, which – to leave to the students to develop on their own, and which to develop together with them, as a research team.

Escherization is found to be a very suitable strategy to motivate the introduction of important mathematical notions, such as the geometric transformations symmetry, translation, rotation and optical reflection, to young students. And, of course, tiling is fun, a never-ending source of creativity, allowing many students to share Escher's enthusiasm that the tiling is the richest source of inspiration ever struck.

Furthermore, tiling is a good example of mathematics as a field of explorations and open problems. Today, the tiling industry in mathematics is one of phenomenal growth. Many mathematicians and scientists in seemingly unrelated fields of research have discovered that other problems often translate into tiling problems, and so new vigour and insight have enriched the subject [13].

4. FIRST IMPRESSIONS

4.1. When working with children

The creative aspect of making sense of mathematics is a real confidence builder for young students. When working with 10-12 year old children from the School academy 21 century (Plovdiv and Assenovgrad) and Sts. Cyril and Methodius school (Sofia) we witnessed their great creativity in making compositions out of geometric figures. In a lot of situations they had to apply geometric transformations, to calculate angles and length of segments (often going ahead of the curriculum for the corresponding grade). They were highly motivated to bring to life their own project and even refused hints from their teachers murmuring: I prefer to use MY mind....

When building their compositions the children used two typical programming style – top-down (designing a preliminary sketch and decomposing it in simpler geometric figures) and bottom-up – trying out various combinations of preliminary given geometric elements. In order to get the desired figure they had to harness many properties of the geometric nature. Turning the turtle to angles with different sign brought quite naturally to a new definition of symmetry with respect to an axis. It is interesting to note that lead by their own aesthetics they tended to reduce the number of different shapes used as building blocks. The first two models in Figure 9 are designed by a boy and a girl (11-year-old) when given the task of creating the image of a person by geometric shapes. The rightmost model next to them is created by an 11-year-old girl after a painting by the Bulgarian artist Pencho Balkanski.

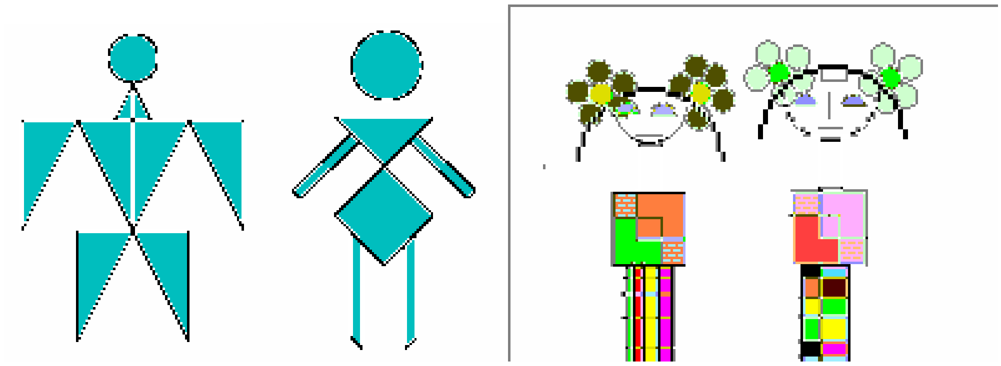

Figure 9. Models created by junior-high-school students

4.2. In teacher education

The curriculum of IV year students in Mathematics and Informatics (to become mathematics and/or informatics teachers) include the Logo based course Informatics in the school mathematics. The goal is to present the students with an educationally rich environment supporting active and exploratory style of learning. We involve students in such activities that they could better understand and experience its educational philosophy and potential. It is in the second part of the course that students get project assignments (individually or in pairs) and work on them on their own, supported by the tutor when necessary. Visual modelling enjoys always a great interest among the students. They often share their enthusiasm in learning to look at abstract art with new eyes after having worked on a program generating variations of certain famous paintings (Figure 10).

Figure 10. Models in the style of Delaunay created by teachers-might-be

For students to become mathematically powerful, they must be flexible enough to approach situations in a variety of ways and recognise the relationships among different points of view.

The project-based learning approach we use in our work with teachers reveals:

- the richness of strategies and approaches to visual modelling;
- the necessity of using mathematical knowledge (which would additionally motivate the students in acquiring it), and
- the challenge of developing appropriate informatics toolkit for students of a given age.

The educational strategies applied at the university level are dynamic – they are enriched after each new edition of the course with ideas gained as a result from discussions with the students. Here are some opinions expressed by them after finishing the course [14]:

  - Projects bring the spirit of something interesting and diverse in contrast to the monotony of the classical exams...
  - ...not only is one learning from one's own successes and failures but also from those of one's neighbours...We saw a lot of interesting ideas and realisations...

We as teacher educators feel confident that our students know the difference between "knowing that" and "knowing how", or, as psychologists would have said, between the figurative and operative knowledge.

5. CONCLUSIONS

Our experience shows that integrating the learning and creative processes by means of visual modelling could contribute to introducing a new learning style in mathematics education. Such type of activities will sensitise students to looking at not only the art but also at the world around them in a more meaningful way. This naturally leads to building new strategies in teacher education, which could prepare teachers for their changing role of partners in a creative process. If we hope for a real change in teacher education, we should bring the future teachers in the situations in which they would stop thinking about the future (in terms of exams or teaching children) and start experiencing what they are doing as intellectually exciting and joyful in its own right [15].

As for the curriculum we fully agree with the understanding [16] that the informatics-mathematics curriculum is co-emergent and fully affected by the embodied actions and interactions of the students enacting it... It is not in the hardware or software, it is not simply in the heads of the students or the teachers – like all knowing it is in the interactions themselves.

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